COSMOGRAPHIC EVALUATION OF DECELERATION PARAMETER USING SNe Ia DATA

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ABSTRACT

The apparent magnitude-redshift data of SNe Ia call for modifications in the standard model energy densities. Under the circumstance that this modification cannot be limited to the addition of a mere cosmological constant, a serious situation has emerged in cosmology, in which the energy densities in the universe have become largely speculative. In this situation, an equation of state of the form $p = w\rho$ itself is not well-motivated. In this paper, we argue that the reasonable option left is to make a model-independent analysis of SNe data, without reference to the energy densities. In this basically kinematic approach, we limit ourselves to the observationally justifiable assumptions of homogeneity and isotropy; i.e., to the assumption that the universe has a RW metric. This cosmographic approach is historically the original one to cosmology. We perform the analysis by expanding the scale factor into a polynomial of order 5, which assumption can be further generalised to any order. The present expansion rates h, q_0, r_0 etc. are evaluated by computing the marginal likelihoods for these parameters. These values are relevant, since any cosmological solution would ultimately need to explain them.

Subject headings: cosmological parameters - cosmology: observations - cosmology: theory - deceleration parameter

1. INTRODUCTION

In the Friedmann models of the universe, the solution of the Einstein's field equation requires an equation of state to relate the energy density and the pressure of the universe (Weinberg 1972). An important recent development in cosmology is that the apparent magnitude-redshift data of SN Ia (Riess *et al.* 1998; Perlmutter *et al.* 1999; Tonry *et al.* 2003; Knop *et al.* 2003) give reasons to suspect that the expansion of the universe is accelerating and thereby points to a major gap

in our understanding of the density components in the universe and the equations of state obeyed by them. In fact, this gap leaves the solution of the Einstein equation speculatory to a great extent. The explanation of all other cosmological observations needs this solution, as it describes the expansion of the background spacetime. Hence these SN data, which are claimed to be the only qualitative signature of an accelerated expansion of the universe (Tonry *et al.* 2003), can be said to put cosmologists back to square one. An important question we need to face at the outset is whether we view this suspected accelerated expansion as a new phenomenon exhibited by the universe in the recent epoch or as a caveat in our understanding of the dynamics of the universe. In this paper, we adopt the position that whereas there is general agreement that the theory needs modifications for the present epoch, theories and observations regarding the past require more careful scrutiny and hence while analysing these data, dependence on specific models needs to be avoided.

Nevertheless, the traditional approach to cosmology is through cosmography (Weinberg 1972), rather than through the cosmological solutions. For the determination of the Hubble parameter H_0, this method is used even now, and the value is considered reliable when one uses only low redshift objects. In order to measure q_0, the present value of the cosmic deceleration parameter, it needs to go to higher redshift objects (such as the brightest galaxy clusters or supernovae). Weinberg (1972) discusses various attempts made in the past in this direction at length. Those measurements used the brightest cluster galaxies as distance indicators, which are poor in reliability, and this led Weinberg to remark that, about the precise value of q_0, the knowledge (in 1972) is as little as that in 1931.

It seems that this interest in the value of q_0 declined, as cosmologists became complacent with the standard model (which is a Friedmann model with either dust or radiation as component). Even after the release of substantial amount of SN Ia data in recent years, though it is generally believed that these data imply an accelerating universe, only very few works are done to evaluate the cosmic deceleration parameter in any truly model-independent way, i.e., without making any assumptions on the energy densities etc. Instead, most analyses of SN Ia data aim at defending the standard cosmological solutions or their variants, which are obtained by way of introducing exotic energy densities with strange equations of state and so on. When there is no clear prescription for ρ, the density of the universe, even an equation of state of the form $p = w\rho$ is not well-motivated. It is unfortunate that some of the SN data available in the literature itself is in a form suitable only to specific models.

However, some exceptions to this trend exist (Trentham 2001; Daly & Djorgovski 2003; Wang & Mukherjee 2003; Daly & Djorgovski 2004). Trentham (2001) examines the effects of various assumptions we make in evaluating the cosmic acceleration, such as the large scale isotropy and homogeneity, flatness, etc. and concludes that failures of two or more such assumptions in concordance may have strong effects. Daly and Djorgovski (2003) aim at a model-independent way to compute the deceleration parameter q as a function of redshift z. They divide the data points into redshift intervals for which a dimensionless coordinate distance $y(z)$ is fitted in the form of

second order polynomials. These are then numerically differentiated to obtain $q(z)$. An attempt is also made to find the value of the redshift at which the speculated transition from deceleration to acceleration occurred. Wang and Mukherjee (2004) allow the dark energy density to be an arbitrary function of the redshift z and present a model independent reconstruction of the dark energy density and have found that it varies with time at less than 2σ.

This paper aims at evaluating q_0 cosmographically, without making any kind of assumptions regarding its energy content. The method requires the least amount of speculatory inputs in the analysis of the supernova data. We confine ourselves to the assumptions that the universe has a Robertson-Walker (RW) metric with $k = \pm 1$ or 0 and that the scale factor can be approximated by the first six terms in a Taylor series; i.e., as a fifth order polynomial. (In (Weinberg 1972), only the first three terms are taken. As we shall see later, the complexity increases with the number of terms taken, which will require high computing power to deal with.) The coefficients in this expansion are considered as the parameters of the theory. The only other parameter, which does not come into this expansion is M, the absolute luminosity of the standard candles. We evaluate the likelihood for these parameters for all possible combinations of parameter values. The data used are that of the 54 'All SCP SNe' reported in (Knop *et al.* 2003). For the parameters of interest, we find the marginal likelihood by marginalising over all other parameters. This method has the advantage that while giving the likelihood for one parameter, it takes care of the uncertainties in the other ones.

The method is simple and is a straight-forward generalisation of the one discussed in (Weinberg 1972). What is presented here are some first results, which can be improved substantially with the fastness of computation. This method, which can be described as basically kinematic, gives valuable information regarding the expansion rates of the universe and provides a testing ground for all cosmological solutions.

2. THE METHOD

To start with, we state our assumption explicitly: the universe has a RW metric, with $k = \pm 1$ or 0. Then the scale factor can be expanded into a Taylor series

$$a(t) = a_0 \left[1 + \frac{a_0^{(1)}}{a_0}(t - t_0) + \frac{a_0^{(2)}}{2!a_0}(t - t_0)^2 + \frac{a_0^{(3)}}{3!a_0}(t - t_0)^3 + \frac{a_0^{(4)}}{4!a_0}(t - t_0)^4 + \frac{a_0^{(5)}}{5!a_0}(t - t_0)^5 + ... \right]. \tag{1}$$

Henceforth, we limit ourselves by keeping only up to the fifth order term in the above series and assume that it is a good approximation. Here $a_0^{(i)}$ refers to the present value of the i^{th} derivative of the scale factor a with respect to time. We substitute

$$\frac{a_0^{(1)}}{a_0} \equiv H_0 \equiv a_{(1)}, \qquad \frac{a_0^{(2)}}{2!a_0} \equiv -\frac{q_0 H_0^2}{2!} \equiv a_{(2)}, \qquad \frac{a_0^{(3)}}{3!a_0} \equiv \frac{r_0 H_0^3}{3!} \equiv a_{(3)}, \qquad \frac{a_0^{(4)}}{4!a_0} \equiv -\frac{s_0 H_0^4}{4!} \equiv a_{(4)},$$

$$\frac{a_0^{(5)}}{5!a_0} \equiv \frac{u_0 H_0^5}{5!} \equiv a_{(5)}, \tag{2}$$

and

$$t - t_0 \equiv T \tag{3}$$

to get

$$\begin{aligned} a(t_0+T) &= a_0\left[1 + H_0 T - \frac{q_0 H_0^2}{2!}T^2 + \frac{r_0 H_0^3}{3!}T^3 - \frac{s_0 H_0^4}{4!}T^4 + \frac{u_0 H_0^5}{5!}T^5\right] \\ &= a_0\left[1 + a_{(1)}T + a_{(2)}T^2 + a_{(3)}T^3 + a_{(4)}T^4 + a_{(5)}T^5\right]. \end{aligned} \tag{4}$$

Now one can write

$$\frac{1}{a(t_0+T)} = \frac{1}{a_0}\left(1 + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4 + \kappa T^5 + \mu T^6 + ...\right), \tag{5}$$

where

$$\beta = -a_{(1)}, \qquad \gamma = -a_{(1)}\beta - a_{(2)}, \qquad \delta = -a_{(1)}\gamma - a_{(2)}\beta - a_{(3)}, \qquad \epsilon = -a_{(1)}\delta - a_{(2)}\gamma - a_{(3)}\beta - a_{(4)},$$

$$\kappa = -a_{(1)}\epsilon - a_{(2)}\delta - a_{(3)}\gamma - a_{(4)}\beta - a_{(5)}$$

and

$$\mu = -a_{(1)}\kappa - a_{(2)}\epsilon - a_{(3)}\delta - a_{(4)}\gamma - a_{(5)}\beta, \qquad \text{etc.} \tag{6}$$

Even though our assumption is that of a fifth order polynomial for the scale factor, the series (5) cannot be terminated anywhere without proper checking of the remainder term. We have kept terms up to order 6 in T for better accuracy. This can still be improved if the need arises.

For a $k = 0$ RW metric,

$$r_1 = \int_{t_1}^{t_0} \frac{c\,dt}{a(t)} = \frac{c}{a_0}\int_{T_1}^{0}\left(1 + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4 + \kappa T^5 + \mu T^6 + ...\right) dT \tag{7}$$

(where c is the velocity of light), or

$$r_1 a_0 = c\left(-T_1 - \frac{\beta T_1^2}{2} - \frac{\gamma T_1^3}{3} - \frac{\delta T_1^4}{4} - \frac{\epsilon T_1^5}{5} - \frac{\kappa T_1^6}{6} - \frac{\mu T_1^7}{7} - ..\right). \tag{8}$$

Similarly for the $k = \pm 1$ cases, we have

$$r_1 a_0 = a_0\ S_k\left[\frac{c}{a_0}\left(-T_1 - \frac{\beta T_1^2}{2} - \frac{\gamma T_1^3}{3} - \frac{\delta T_1^4}{4} - \frac{\epsilon T_1^5}{5} - \frac{\kappa T_1^6}{6} - \frac{\mu T_1^7}{7} - ..\right)\right], \tag{9}$$

where $S_k(x) = \sin x$ for $k = +1$ and $S_k(x) = \sinh x$ for $k = -1$. However, since the $k = 0$ case is identical to $a_0 \gg 1$, we consider only the $k = \pm 1$ cases. We invert the following equation

$$1 + z = \frac{a(t_0)}{a(t_0 + T_1)} = 1 + \beta T_1 + \gamma T_1^2 + \delta T_1^3 + \epsilon T_1^4 + \kappa T_1^5 + \mu T_1^6 + ... \tag{10}$$

numerically to obtain T_1 in terms of z and substitute in the above equation for $r_1 a_0$ and compute

$$D = r_1 a_0 (1 + z), \tag{11}$$

$$m = 5 \log\left(\frac{D}{1\mathrm{Mpc}}\right) + 25 + M. \tag{12}$$

Here D and hence m are functions of z, a_0, β , γ , δ , ϵ, κ, μ and M (or equivalently z, a_0, H_0, q_0, r_0, s_0, u_0, k, and M).

The likelihood function is

$$\mathcal{L} = \exp(-\chi^2(a_0, h, q_0, r_0, s_0, u_0, k, M)/2). \tag{13}$$

The marginal likelihood L (Drell 2000; Riess *et al.* 1998; John & Narlikar 2002) for each of h, q_0, r_0, s_0 and u_0 are found by assuming flat priors for the remaining parameters in the set a_0, h, q_0, r_0, s_0, u_0, k, and M, in the corresponding intervals Δa_0, Δh, Δq_0, Δr_0, Δs_0, Δu_0, Δk, and ΔM as the case may be. For example, $L(q_0)$ is found as

$$L(q_0) = \frac{1}{2}\sum_{k=-1,1} \frac{1}{\Delta a_0}\frac{1}{\Delta h}\frac{1}{\Delta r_0}\frac{1}{\Delta s_0}\frac{1}{\Delta u_0}\frac{1}{\Delta M}\int e^{-\chi^2/2} da_0\ dh\ dr_0\ ds_0\ du_0\ dM. \tag{14}$$

The factor outside the integral is the product of flat priors. For our purpose, the value of this is uninteresting.

3. COMPUTATION OF LIKELIHOOD FUNCTIONS

The data analysed are that found in (Knop *et al.* 2003), in Tables 3, 4, and 5 and are reproduced in Table 1 of this paper. These 54 SNe Ia belong to the subset ‘All SCP SNe Ia’, mentioned in

Table 8 of (Knop *et al.* 2003). But the data we are using are slightly different from that used by these authors for cosmological fits. The apparent magnitudes used here are stretch corrected, which relieves us of the one parameter α in the calculations.

An important part of the computation is the numerical integration to obtain the marginal likelihood. Another part is the numerical solution of equation (10) for the cosmic time T_1, given the redshift z, for each set of parameter values. The results for these numerical solutions are cross-checked and found that they are reliable. However, provisions were left to discard those parameter values for which the accuracy is poor (i.e., when the solution for T_1 is such that the difference between the left and right sides of the equation is greater than 0.001). In another part, we find the reciprocal of the polynomial for $a(t)$ [see equation (5)] numerically. This procedure is prone to more serious errors due to the truncation of the series. In order to minimise it, we have kept seven terms on the right hand side. We cross-check also this calculation and discard those parameter values which do not give sufficient accuracy for $1/a(t)$, as in the previous case.

The resulting curves L are shown in Figures 1 - 4. The mean value of q_0 obtained from Figure 2 is $< q_0 >\approx -0.77$. Here the ranges of the parameters used in the integral are $3000 < a_0 < 8000$, $0.6 < h < 0.8$, $-15 < r_0 < 15$, $-65 < s_0 < 65$, $-150 < u_0 < 150$, $-1 < k < 1$, and $-19.6 < M < -19.1$, where the step sizes are $\delta a_0 = 1000$, $\delta h = 0.04$, $\delta r_0 = 0.5$, $\delta s_0 = 1$, $\delta u_0 = 25$, $\delta k = 2$, and $\delta M = 0.1$. The parameter M has reasonable prior, which we have employed in the above case. The prior range $a_0 > 3000$ Mpc is also reasonable and the upper limit (8000 Mpc) is chosen high enough to incorporate flat ($k = 0$) models indirectly. In order to ascertain the validity of the priors for the parameters h, r_0, and s_0, the marginal likelihoods for these parameters are computed and are given in Figures 1, 3 and 4. These curves show that the contributing regions of these parameters are well within the ranges we have employed.

The mean values of the other parameters are the following: $< h >\approx 0.67$, $< s_0 >\approx -2.67$, and $< r_0 >\approx 2.64$. The marginal likelihood for h clearly shows that it is not possible to narrow down the estimation of this parameter, even with such refined and high redshift data.

4. DISCUSSION

What we present here is just the revival of an old method of analysing the apparent magnitude-redshift data. With the availability of modern high performace computers, this becomes a natural choice. Under the circumstance that there is no clear prescription for the kind and number of energy densities present and no definite motivation for an equation of state of the form $p = w\rho$, perhaps, the only option left to understand the dynamics of the universe from SNe data is to evaluate the expansion rates such as H_0, q_0, r_0, etc., as we have done here. The rationale behind the method is straight-forward and well-established. It is limited only for the perfection of the computations involved. The calculation of $L(q_0)$ took nearly 75 hours on using a Pentium 4 Linux machine. However, the accuracy can still be improved with the use of more advanced computational

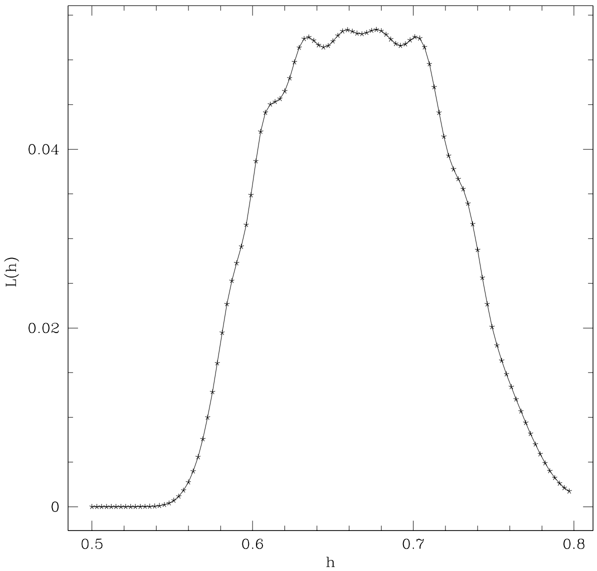

Fig. 1.— The marginal likelihood for h.

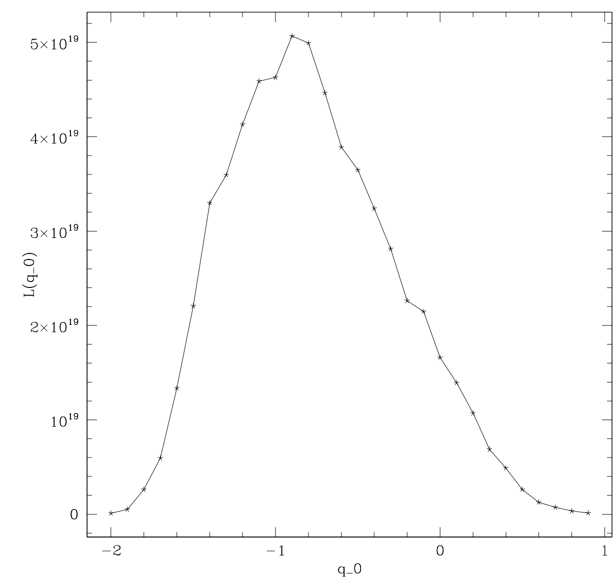

Fig. 2.— The marginal likelihood for q_0.

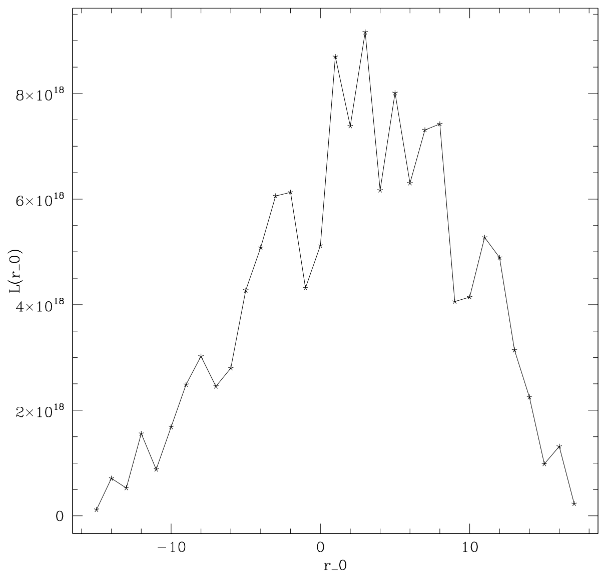

Fig. 3.— The marginal likelihood for the parameter r_0.

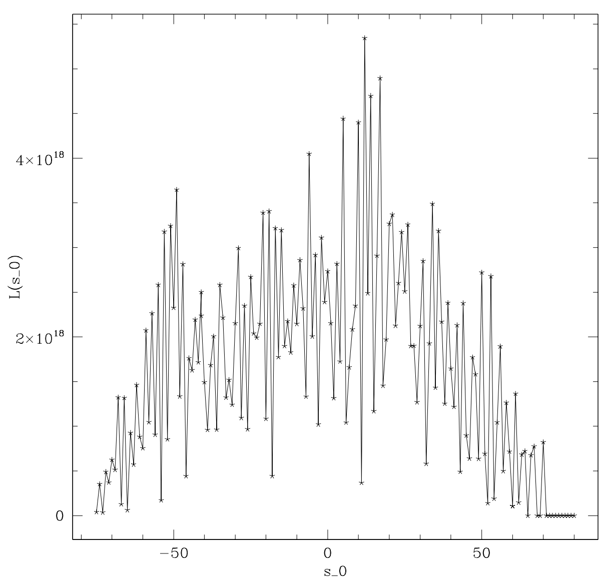

Fig. 4.— The marginal likelihood for the parameter s_0.

facilities.

A potential source of error in the present cosmographic approach is that the perturbative expansion of a(t) used in this paper necessitates truncations; these result in the breaking down of the approximations for certain parameters, which are then left out. This leads to fluctuations in the likelihood functions of higher derivative expansion rates like r_0 and s_0 (see Figs. 3-4). It is found that if we use less number of terms in the perturbative expansion (for example, a fourth order polynomial for $a(t)$), even $L(q_0)$ exhibits some gaps, eventhough its envelope and the mean value $< q_0 >\approx -0.76$ calculated therefrom is nearly the same as that in the present case ($< q_0 >\approx -0.77$). Thus it is reasonable to expect that the fluctuations in $L(r_0)$ and $L(s_0)$ in Figures 3 and 4 will disappear, while their envelopes and mean values may remain mostly unchanged, if higher order polynomials than that used in this paper are employed. (But it should be noted that the addition of one more term will increase the computation time at least by an order of magnitude.) However, since our primary concern is the evaluation the deceleration parameter q_0, we only want to ascertain that the contributiong regions of r_0, s_0 etc. are $-15 < r_0 < 15$, $-65 < s_0 < 65$ etc., which regions we have chosen in our calculations. A similar exercise is performed also for u_0. It is thus presumed that Figures 3 - 4 serve their purpose.

Any error in the numerical integration can be reduced by decreasing the step sizes δq_0, δh, etc. While finding $1/a(t)$ in equation (5), we can add more terms to the right hand side to get more accurate results. In the numerical solution of equation (10) for T_1 in terms of z by the Newton-Raphson method, the number of iterations may be increased, but even in the present case, it does not need to go beyond $i = 10$.

This method is complementary to the other analyses of SN data mentioned in the introduction. The complementarity of this approach to those of others arises from the fact that it is model-independent and perturbative. The advantage of evaluating the marginal likelihood function is that it automatically takes care of the uncertainties in the other parameters. (The marginalisation technique used is the same as that used to eliminate the combination of Hubble parameter and absolute magnitude of SN Ia by Riess et.al. (Riess *et al.* 1998)). Also, while evaluating $L(q_0)$ and other likelihoods, we use the entire SN data set to estimate the value of the single parameter at the present epoch, and hence our evaluation of q_0 can be considered reliable.

Our cosmographic method encompasses all models which have the RW metric and hence it is more general than the ones used for Friedmann models. The evaluation of $L(q_0)$ is not by restricting ourselves to any specific equation of state or to the number of choices of different possible energy densities. The mean value of q_0 shows that the universe is indeed accelerating. Evaluation of all the expansion rates like q_0, r_0, etc. is important because it would ultimately become mandatory for every viable cosmological models to explain the values we have obtained for these parameters.

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Table 1: SN DATA

SN	z	m	error
1997ek	0.863	24.59	0.19
1997eq	0.538	23.15	0.18
1997ez	0.778	24.41	0.18
1998ay	0.638	23.92	0.19
1998ba	0.430	22.90	0.18
1998be	0.644	23.64	0.18
1998bi	0.740	23.85	0.17
2000fr	0.543	23.16	0.17
1995ar	0.465	23.35	0.22
1995as	0.498	23.74	0.23
1995aw	0.400	22.57	0.18
1995ax	0.615	23.38	0.22
1995ay	0.480	22.90	0.19
1995az	0.450	22.66	0.20
1995ba	0.388	22.60	0.18
1996cf	0.570	23.30	0.18
1996cg	0.490	23.11	0.18
1996ci	0.495	22.78	0.18
1996cl	0.828	24.49	0.46
1996cm	0.450	23.11	0.18
1997F	0.580	23.57	0.20
1997H	0.526	23.09	0.19
1997I	0.172	20.29	0.17
1997N	0.180	20.48	0.17
1997P	0.472	22.99	0.18
1997Q	0.430	22.52	0.17
1997R	0.657	23.80	0.19
1997ac	0.320	21.96	0.17
1997af	0.579	23.38	0.18
1997ai	0.450	22.63	0.22
1997aj	0.581	23.16	0.18
1997am	0.416	22.63	0.18
1997ap	0.830	24.38	0.18
1990O	0.030	16.33	0.20
1990af	0.050	17.39	0.18
1992P	0.026	16.14	0.19
1992ae	0.075	18.35	0.18
1992al	0.014	14.42	0.23
1992aq	0.101	19.12	0.17
1992bc	0.020	15.18	0.20
1992bg	0.036	16.66	0.20
1992bh	0.045	17.64	0.18
1992bl	0.043	17.03	0.18
1992bo	0.018	15.42	0.21
1992bp	0.079	18.16	0.18
1992bs	0.063	18.26	0.18
1993B	0.071	18.40	0.18
1993O	0.052	17.53	0.18
1994M	0.024	16.07	0.20
1994S	0.016	14.83	0.22
1995ac	0.049	17.17	0.18
1996C	0.030	16.74	0.19
1996ab	0.125	19.47	0.19
1996bl	0.035	16.71	0.19